International Journal of Research in Advent Technology, Vol.6, No.10, October 2018
E-ISSN: 2321-9637
Available online at www.ijrat.org

Applications of Mohand Transform to Mechanics and Electrical Circuit Problems

P.Senthil Kumar[1], P.Gomathi[2], S.Gowri[3], A.Viswanathan[4]
Professor, Department of Mathematics, SNS College of Technology, Coimbatore-641035, TN, India[1,4]
Assistant Professor, Department of Mathematics, SNS College of Technology, Coimbatore-641035, TN, India[2,3]
psenthil3@gmail.com[1], *gomathisenthilkumar2011@gmail.com*[2], *gowrisathasivam@gmail.com*[3], *deansandh@snsct.org*[4]

Abstract-In this paper a new integral transform namely Mohand transform was applied to solve the ordinary differential equations in mechanics and electrical circuit problems with initial conditions.

Keywords-Mohand transform, differential equations.

1. INTRODUCTION

In order to solve differential equations, several integral transforms were extensively used and applied in theory and application such as the Laplace [2], Fourier, Mellin, Hankel, Sumudu [3], Elzaki [4] and Aboodh [5] transforms. Recently Mohand transform was introduced by Mohand Mahgoub to facilitate the process of solving ordinary and partial differential equations in the time domain. Many of the engineering, physics, biology, chemistry applications, integral equations play a vital role. P.Senthil Kumar and A. Viswanathan [6] proposed Mahgoub transform to solve the mechanics and electrical circuit problems. P. Senthil Kumar and S. Vasuki [7] proposed Aboodh transform to solve mechanics and electrical circuit problems. In this study, our purpose is to show the applicability of this interesting new transform and its efficiency in solving the differential equations.

2. MOHAND TRANSFORM

The Mohand transform [1] is defined for the function of exponential order. We consider functions in the set A defined by

$$A=\left\{ f(t):\exists M,k_1,k_2>0,\left|f(t)\right|<Me^{\frac{|t|}{k_j}},\ if\ \ t\in(-1)^j\times[0,\infty)\right\}$$

where the constant M must be finite number k_1, k_2 may be finite or infinite.

The Mohand transform denoted by the operator M [.] defined by the integral equation

$$M[f(t)]=R(v)=v^2\int_0^\infty f(t)e^{-vt}dt,\ t\geq 0,\ k_1\leq v\leq k_2 \_\_(1).$$

2.1. *Some standard functions*

Mohand transform of some standard functions are listed in table 1.

Table 1. Standard functions

S.No.	f(t)	*M* [*f(t)*]
1.	1	v
2.	t	1
3.	t^n	$\frac{n!}{v^{n-1}}$
4.	e^{at}	$\frac{v^2}{v-a}$
5.	e^{-at}	$\frac{v^2}{v+a}$
6.	$cosat$	$\frac{v^3}{v^2+a^2}$
7.	$sinat$	$\frac{v^2}{v^2+a^2}$
8.	$coshat$	$\frac{v^3}{v^2-a^2}$
9.	$sinhat$	$\frac{v^2}{v^2-a^2}$

2.2. *Transform of derivatives*

If M [$f(t)$] = $R(v)$ then

a) $M[f'(t)]=vR(v)-v^2f(0)$

b) $M[f''(t)] = v^2 R(v) - v^3 f(0) - v^2 f'(0)$

c) $M[f'''(t)] = v^3 R(v) - v^4 f(0) - v^3 f'(0) - v^2 f''(0)$

In general

$M[f^n(t)] = v^n R(v) - v^{n+1} f(0) - v^n f'(0) - v^{n-1} f''(0) - ...$

That is, $M[f^n(t)] = v^n R(v) - \sum_{k=0}^{n-1} v^{n-k+1} f^{(k)}(0)$

3. APPLICATIONS TO MECHANICS

3.1. *Application 1*

A particle P of mass 2 grams moves on the X-axis and is attracted towards origin O with a force numerically equal to $8X$. If it is initially at rest at X=10, find its position at any subsequent time assuming

(a) No other force acts

(b) A damping force numerically equal to 8 times the instantaneous velocity acts.

Solution: (a) From Newton's law, the equation of motion of the particle is

$$2\frac{d^2X}{dt^2} = -8X \quad (or) \qquad \frac{d^2X}{dt^2} + 4X = 0 \_\_\_\_\_ (2)$$

with the initial conditions $X(0) = 10$ and $X'(0) = 0$.

Taking the Mohand transform of both sides of "Eq.(2)", we have

$$M[X''] + 4M[X] = 0$$

Using Mohand transform of derivatives, we get

$$v^2 M[X] - v^3 X(0) - v^2 X'(0) + 4M[X] = 0$$

Applying the initial conditions, then

$$v^2 M[X] - 10v^3 + 4M[X] = 0$$

$$M[X]\{v^2 + 4\} = 10v^3$$

$$M[X] = 10\left(\frac{v^3}{v^2+4}\right)$$

Take inverse Mohand transform, we get

$$X = 10\, M^{-1}\left(\frac{v^3}{v^2+4}\right) = 10\cos 2t .$$

(b) In this case, the equation of motion of particle is

$$2\frac{d^2X}{dt^2} = -8X - 8\frac{dX}{dt} \text{ (or) } \frac{d^2X}{dt^2} + 4\frac{dX}{dt} + 4X = 0 \_\_\_\_ (3)$$

with initial conditions $X(0) = 10$ and $X'(0) = 0$.

Taking the Mohand transform of both sides of "Eq.(3)", we have

$$M[X''] + 4M[X'] + 4M[X] = 0 .$$

Using Mohand transform of derivatives, then

$$\left\{v^2 M[x] - v^3 X(0) - v^2 X'(0)\right\} + 4\left\{vM[X] - v^2 M[0]\right\} + 4M[X] = 0$$

Apply the initial conditions, then we have

$$M[X] = \frac{10v^3 + 40v^2}{(v+2)^2} = \frac{10v^2}{(v+2)} + \frac{20v^2}{(v+2)^2}$$

Now take the inverse Mohand transform, then we get

$$X = 10M^{-1}\left(\frac{v^2}{(v+2)}\right) + 20M^{-1}\left(\frac{v^2}{(v+2)^2}\right)$$

$$X = 10e^{-2t} + 20te^{-2t} .$$

4. APPLICATIONS TO ELECTRIC CIRCUITS

The Mohand transform can also be used to determine the charge on the capacitors and currents as functions of time.

4.1. *Applicatioin 1*

An alternating e.m.f Esinwt is applied to an inductance L and a capacitance C in series. Find the current in the circuit.

Solution: The differential equation for the determination of the current I in the circuit is given as (since R=0)

$$L\frac{dI}{dt} + \frac{Q}{C} = E\sin\omega t \_\_\_\_\_\_ (4) \text{, where } I = \frac{dQ}{dt} \_\_(5)$$

Also at $t = 0,\ I = 0 = Q$.

Taking Mohand transform of both sides of "Eq.(4)" and "Eq.(5)", we have

$$(4) \Rightarrow M[I'] + n^2 M[Q] = \frac{E}{L} M[\sin\omega t], \text{ where } \frac{1}{LC} = n^2$$

Using the Mohand transform of derivatives, then

$$v\,M[I] - v^2 I[0] + n^2 M[Q] = \frac{E}{L}\left(\frac{\omega v^2}{v^2+\omega^2}\right)$$

Apply initial condition, then

$$\nu M[I] + n^2 M[Q] = \frac{E}{L}\left(\frac{\omega\nu}{\nu^2+\omega^2}\right) \qquad (6)$$

(5) $\Rightarrow$ $M[I] = M[Q'] = \nu M[Q] - \nu^2 Q(0)$

$$M[I] = \nu M[Q] \qquad (7)$$

From "Eq.(6)" and "Eq.(7)", we get

$$M[I] = \nu^3 \frac{E\omega}{L}\left(\frac{n^2}{(n^2+\nu^2)(\nu^2+\omega^2)}\right)$$

Take inverse Mohand transform, we get

$$I = \nu^3 \frac{E\omega}{L} M^{-1}\left(\frac{n^2}{(n^2+\nu^2)(\nu^2+\omega^2)}\right).$$

$$I = \frac{E\omega}{L(n^2-\omega^2)}\left(\cos\omega t - \cos nt\right)$$

4.2. *Application 2*

Solve : $L\frac{dx}{dt} + Rx = Ee^{-at}$ given $x(0) = 0$.

Solution: Given equation is $\frac{dx}{dt} + \frac{R}{L}x = \frac{E}{L}e^{-at}$ ____(8)

Take Mohand transform of both sides of "Eq.(8)", we get

$$M[x'] + \frac{R}{L}M[x] = \frac{E}{L}M[e^{-at}]$$

$$\nu M[x] - \nu^2 x(0) + \frac{R}{L}M[x] = \frac{E}{L}M\left[\frac{\nu^2}{\nu+a}\right]$$

$$M[x] = \frac{E\nu^2}{(\nu+a)(\nu L+R)}$$

Take inverse Mohand transform, we get

$$x = M^{-1}\left(\frac{E\nu^2}{(\nu+a)(\nu L+R)}\right)$$

$$x = \frac{E}{R-La}\left[e^{-at} - e^{-\frac{Rt}{L}}\right].$$

5. CONCLUSION

In this paper, we have successfully applying the new integral transform Mohand transform for solving ordinary differential equations in Mechanics and electrical circuit problems. The results are exact and also verified.

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